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Relationships between Median Values and between Aleatory Variabilities for Different Definitions of the Horizontal Component of Motion

by Katrin Beyer and Julian J. Bommer

Abstract Ground-motion prediction equations (GMPE) for horizontal peaks of acceleration and velocity, and for horizontal response spectral ordinates, have employed a variety of definitions for the horizontal component of motion based on different treatments of the two horizontal traces from each accelerogram. New definitions have also recently been introduced and some of these will be used in future GMPEs. When equations using different horizontal-component definitions are combined in a logic-tree framework for seismic-hazard analysis, adjustments need to be made to both the median values of the predicted ground-motion parameter and to the associated aleatory variability to achieve compatibility among the equations. Because there is additional aleatory variability in the empirical ratios between the median values for different components, this uncertainty also needs to be propagated into the transformed logarithmic standard deviation of the adjusted equations. This study provides ratios of both medians and standard deviations for all existing component definitions with respect to the geometric mean of the two horizontal components, which is currently the most widely used in prediction equations. The standard deviations on the ratios of the medians are also reported. This article also discusses the issue of the ratios of different horizontal component definitions in relation to the specification of seismic input for dynamic structural analyses, highlighting the importance of consistency between the component definition used to derive the elastic design-response spectrum and the way that biaxial dynamic loading input is prepared.

Introduction

In developing equations for the prediction of horizontal ground-motion parameters, whether these be the peaks of acceleration and velocity or response spectral ordinates, a decision needs to be made regarding how to treat the two horizontal components of each recorded accelerogram. Different options of treating the two horizontal components have been employed in the literature, none of which can be identified as optimal for all applications. The key issue is to ensure that the selected definition is used consistently at all stages, from derivation of the ground-motion prediction equations through to generation of the design-response spectrum and its application in structural analysis. The definition of the horizontal component of motion used in the ground-motion prediction equation becomes particularly important when structural analysis is carried out considering seismic loading in two perpendicular horizontal directions, which in general, will coincide with the axes of the building. For dynamic structural analysis, the input is required in the form of acceleration time histories, which in general, will be scaled to match, in some period range and to some specified criteria, the elastic design-response spectrum. In this situation, it is essential to correctly identify the horizontal-component definition employed to derive the response spectrum so that the scaling can correctly preserve the relationship between the two horizontal components of the accelerogram. For these reasons it is useful to establish the ratios between the values of different ground-motion parameters so that appropriate adjustments can be made when scaling records, either to the accelerogram or to the target spectrum.

Relationships between the median values of horizontal motions and between their aleatory variabilities are also required to achieve compatibility among ground-motion prediction equations using different definitions when these are combined in a logic-tree framework for seismic-hazard analysis (Bommer *et al.*, 2005). In each case, the aleatory variability associated with the empirically derived adjustments must be propagated into the aleatory variability associated with the predictive equation. Several adjustments can be necessary to compensate for different definitions of both the predicted and explanatory variables used in the equations, including magnitude scale and distance metric. For the latter, Scherbaum *et al.* (2004) have derived distributions for the relationships between different measures of the source-to-site distance using randomly distributed receivers. Scherbaum *et al.* (2005) examine the impact of all of the adjustments on the converted ground-motion models and demonstrate that distance conversion has the greatest impact on the median values of the parameter, in particular, at short distances. The impact on the aleatory variability of applying adjustments for different distance metrics is also very high because of the very large differences that can exist between pairs of metrics, especially for stations close to the earthquake source. If the seismic sources are faults of known geometry, then the problem is obviated because each equation can be employed with its own distance definition. Although computationally intensive, the necessity for the distance conversions for area sources can also be avoided by simulating individual earthquake sources within source zones in such a way as to allow the source-to-site distance to be calculated according to the definition used in each predictive equation. The large penalty on the sigma value can thus be avoided, but not so for the magnitude and component conversions. The standard deviation associated with empirical relationships between different magnitude scales is generally of the order of 0.2 units. Therefore the penalty paid by applying magnitude-scale conversions is to increase values of the standard deviation on the base-10 logarithm of the ground-motion parameter from 0.25 to 0.256 for peak ground acceleration (PGA) and from 0.32 to 0.335 for the spectral acceleration at 1.0 sec, using for illustrative purposes the equations of Ambraseys *et al.* (1996). Although these increases may not appear to be very large, the impact on the seismic hazard can be significant, in particular, when low annual frequencies of exceedance are considered. For this reason, as well as estimating median ratios between the median values of ground-motion parameters obtained using different horizontal-component definitions, it is also necessary to estimate the standard deviations associated with these ratios to correctly propagate the uncertainty.

This article presents a derivation of ratios between median values and between aleatory variabilities for a large number of horizontal-component definitions. This article is effectively an extension of the brief treatment of the same subject presented in Bommer *et al.* (2005), with many more definitions of the horizontal component of motion, because the earlier article only addressed those definitions that were used in the particular equations combined in the logic-tree analysis performed therein for illustrative purposes. The parameters for which the ratios are calculated are 5%-damped spectral accelerations for 77 response periods from 0.01 to 5.0 sec, PGA and peak ground velocity (PGV). The dataset employed for this analysis consisted of 949 records from the Next Generation of Attenuation (NGA) database (PEER, 2005). The NGA database contains records from shallow crustal earthquakes that originate mainly from the western United States and Taiwan, with some records coming from other active zones such as Turkey. The criteria for the selection were the following:

- Records from the Chi-Chi earthquake in 1999 or from any of its aftershocks were excluded to avoid possible bias due to an over-representation of the Chi-Chi sequence, which contributes more than 50% of the total of 3551 records in the NGA database.
- Records with PGA smaller than 0.05*g* were excluded to focus on motions that are of engineering significance and to avoid problems with resolution of analog records. The PGA was in this case defined as the geometric mean of the maximum acceleration of the *x* and *y* components, as recorded (i.e., longitudinal and transverse).
- Records with a maximum usable period of less than 0.5 sec were excluded. Each remaining record was only used up to its maximum usable period, which is specified in the summary file accompanying the NGA database.
- Earthquakes for which the hypocentral depth was not specified were excluded because the hypocentral depth is used as a parameter in the regression analysis.

The records in the dataset come from 103 different earthquakes that contribute between 1 and 138 accelerograms. The records represent a wide range of different characteristics such as magnitude, distance, rupture mechanism, site class, and instrument type:

- Magnitude and distance. The magnitude-distance distribution is shown in Figure 1, which also indicates the contribution of records from different National Earthquake Hazards Reduction Program (NEHRP) site classes to the dataset.
- Source mechanisms. 333 records from 51 earthquakes with a strike-slip mechanism; 36 records from 12 normal-

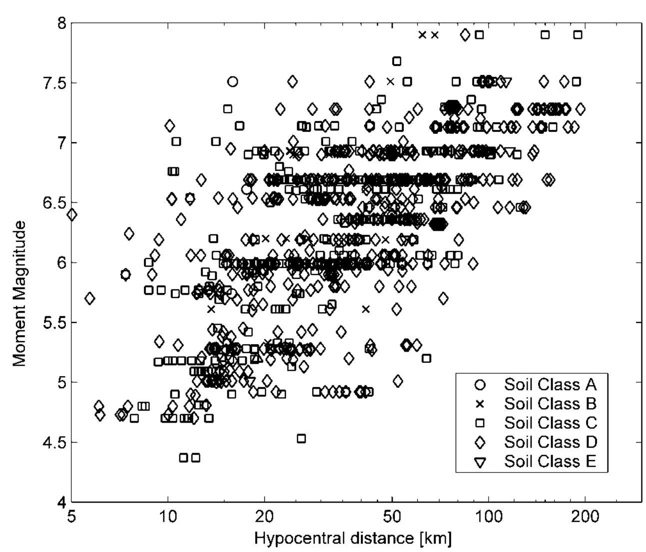

Figure 1. Distribution of records in dataset with respect to moment magnitude, hypocentral distance, and site class.

faulting earthquakes; 329 records from 21 reverse-faulting earthquakes; 223 records from 9 reverse-oblique earthquakes; 25 records from 7 normal-oblique earthquakes; 3 records from 3 earthquakes with undefined mechanism.
- NEHRP site classes: 8 records from site class A; 37 records from site class B; 358 records from site class C; 534 records from site class D; 11 records from site class E; 1 record where the site was not specified.

In the next section the different definitions of the horizontal component of motion employed in this study are defined. Then, the steps involved in transforming a ground-motion prediction equation (GMPE) derived for one component definition to another are presented and discussed in the third section. The ratios of median values and of aleatory variabilities derived from this dataset are given in the fourth and fifth sections of this article respectively. This article concludes with a brief overview of the main conclusions and a discussion of the applications of the results to GMPEs and to preparing input records for dynamic structural analysis.

Definitions of the Horizontal Component of Motion

Table 1 summarizes the different definitions of horizontal ground-motion measures considered in this study and in addition, indicates which definitions are widely used in ground-motion prediction equations and which are relevant to the scaling of natural accelerograms for use in dynamic structural analysis. All known definitions are included, for completeness, even though at least two are indicated to be effectively obsolete with respect to GMPEs and irrelevant to dynamic loading for structural analysis. These indications should be borne in mind when interpreting the results and their significance.

The calculation of most of the definitions is very straightforward, with the exception of GMRotD50 and GMRotI50, for which the reader is referred to the companion article by Boore *et al.* (2006), and the principal directions. The principal directions are determined as the set of orthogonal directions for which the cross-correlation $\rho_{xy} = \mu_{xy}/(\sigma_x \sigma_y)$ is zero and the variances σ_i^2 of the two components maximum and minimum, respectively, are:

$$\sigma_x^2 = \frac{1}{T_d}\int_0^{T_d} \left(a_x(t) - \overline{a_x(t)}\right)^2 dt \qquad (1)$$

$$\sigma_y^2 = \frac{1}{T_d}\int_0^{T_d} \left(a_y(t) - \overline{a_y(t)}\right)^2 dt \qquad (2)$$

$$\mu_{xy} = \frac{1}{T_d}\int_0^{T_d} \left(a_x(t) - \overline{a_x(t)}\right)\left(a_y(t) - \overline{a_y(t)}\right) dt \qquad (3)$$

with $a_i(t)$ nonzero mean time history, $\overline{a_i(t)}$ mean value of time history $a_i(t)$ over T_d. The duration T_d here is taken as the entire record length.

The geometric mean is now the most widely used horizontal-component definition in GMPEs. The geometric mean of the spectral values of the x and y components for the period T_i is defined as:

$$Sa_{GMxy}(T_i) = \sqrt{Sa_x(T_i) \cdot Sa_y(T_i)}. \qquad (4)$$

As can be easily demonstrated, this is equal to the anti-log of the arithmetic mean of the logarithms of the accelerations in the two orthogonal directions. Because this is currently the most widely used definition of the horizontal component of motion, it is adopted as the reference definition for calculating ratios in this study.

Conversions of GMPE for Different Horizontal-Component Definitions

GMPEs can be split into two parts, the first one representing the logarithmic mean value of the ground-motion measure and the second one representing the variation from the logarithmic mean:

$$\log Sa_i = \mu_{\log Sa_i} + \varepsilon \cdot \sigma_{\log Sa_i}, \qquad (5)$$

where $\mu_{\log Sa_i}$ represents the expected value of the logarithm of the spectral acceleration (or other ground-motion parameter) and the second term is the aleatory variability associated with the prediction. Commonly, the term representing the aleatory variability is split into $\sigma_{\log Sa_i}$, the standard deviation of $\log Sa_i$ and ε, the number of logarithmic standard deviations above or below the logarithmic mean. The aim of this section is to present a method to convert a GMPE for geometric mean (GM_{xy}) component definition to an equivalent GMPE for a different horizontal-component definition, Sa_i (e.g., the envelope of the x and y components). Hence, the GMPE for Sa_{GM}:

$$\log Sa_{GM} = \mu_{\log Sa_{GM}} + \varepsilon \cdot \sigma_{\mathrm{tot},\log Sa_{GM}} \qquad (6)$$

is known, whereas $\mu_{\log Sa_i}$ and $\sigma_{\mathrm{tot},\log Sa_i}$ need to be determined to derive the GMPE for Sa_i. Note that for simplicity, when used as a subscript herein, GM_{xy} is written simply as GM. The variability is annotated as total variability of the logarithm of the spectral acceleration to point out that different components of uncertainty are considered to contribute to the variance of the ground-motion measure, as discussed at the end of this section.

To transform the GMPE for Sa_{GM} to a GMPE for Sa_i one needs to make assumptions regarding the distributions of these ground-motion measures. In general, it is assumed that the spectral acceleration of a component with an arbitrary orientation is lognormally distributed. Hence the geometric mean of the spectral acceleration of two orthogonal com-

Table 1
Definitions of the Horizontal Component of Motion Considered in This Study

Symbol	Definition	GMPE*	Ratio to GM_{xy}†	DSA‡
x*x*, *y*	Orientation as recorded. Commonly the orientation of the recording instruments is essentially arbitrary with respect to the fault alignment (very often north–south and east–west) and is generally not correlated to the orientation of nearby faults.	√	A	√
FN, *FP*	Fault-normal and fault-parallel components	√§	B	√
Principal 1 Principal 2	Components along principal directions (see text)	×	B	√
AM_{xy}	Arithmetic mean of spectra of *x* and *y* components	×‖	—	×
GM_{xy}	Geometric mean of spectra of *x* and *y* components	√	A	√
Both	Both horizontal components of a record are considered and treated as two independent realizations of a random process. This definition was used, in particular, when ground-motion data were still very sparse (e.g, McGuire, 1977).	√	A	√
Random	Random choice of one horizontal component from each accelerogram	×#	—	√
GMRotD50	This component definition accounts for the random orientation of the horizontal axis system by choosing, at each response period, the median value of the geometric mean from all possible orientations (see the companion paper by Boore *et al.*, 2006). To determine the median value of all possible orientations the components were rotated with a angle increment of 1.0°.	√	B	×
GMRotI50	This ground-motion measure is an approximation of GMRotD50 with a constant axis orientation for all periods, which minimizes the sum of differences between GMRotI50 and GMRotD50 over all considered periods (Boore *et al.*, 2006).	√	B	√
Larger PGA	From the *x* and *y* components, the one with the larger PGA is chosen and used for all response periods.	×**	—	×
Env	Envelope of *x* and *y* spectra: At each period the larger spectral ordinate of the *x* and *y* components is chosen. This is the common understanding of the "larger component" definition.	√	C	×
MaxD	At each period the maximum spectral ordinate from all possible orientations of the horizontal axis system is determined. This definition differs from GMRotD50 only regarding the considered fractile: MaxD is hence defined as the 100th fractile of the spectral ordinates obtained.	√	B	×
MaxI	This ground motion measure is determined following a procedure similar to the one used by Boore *et al.* (2006) but determining an approximation of MaxD instead of GMRotD50 with a constant axis orientation. The objective function for the angle is slightly different from the one specified by Boore *et al.* (2006) because it considers the differences between MaxI and MaxD only for periods greater than 0.5 sec.	√	B	√

*This indicates whether the definition has been widely used in ground-motion prediction equations; a × indicates that the definition is not in common use and therefore the nature of the distribution of the residuals in the ratio with respect to the GM_{xy} component is not classified.

† Classification indicating the degree to which the residual distribution for the ratio of each parameter to GM_{xy} approximates to lognormal: A, effectively exact match, in most cases by definition; B, not exact but a reasonable approximation; C, clearly not a lognormal distribution. Classifications not given for definitions not widely employed in ground-motion prediction equations (see footnote *).

‡ DSA, dynamic structural analysis. Relevance to the preparation (selection and scaling) of accelerograms for application in dynamic analysis. Any definition that is used for predictive equations is relevant in so much as it can define the target elastic response spectrum, but herein relevance specifically means that it can correspond to the treatment of recorded accelerograms as applied in structural analysis.

§This definition is classified as being relevant, although to date it has been used in relatively few prediction equations (e.g., Bray and Rodriguez-Marek, 2004), having been employed rather in empirical adjustments for the effects of rupture directivity (Somerville *et al.*, 1997).

‖The arithmetic mean would properly only be employed in equations derived without the logarithmic transformation. Campbell (1981) is believed to have used the arithmetic mean of the horizontal components of each accelerogram, but this equation has long been superseded and is therefore obsolete.

#In the compendium of PGA and SA prediction equations by Douglas (2003), only two equations are reported to have used this definition: Cornell *et al.* (1979), which is now obsolete, and Spudich *et al.* (1996), which has been superseded by Spudich *et al.* (1999), which abandoned this definition.

**To the knowledge of the authors, this definition has only been employed by Sabetta and Pugliese (1996); although these equations are still in use in Italy, the authors of this article do not believe that this definition will be used by others (certainly its use would not be encouraged) and that it therefore has a limited shelf life.

ponents *x* and *y* with arbitrary orientation is also lognormally distributed. Therefore, the ratio of Sa_x/Sa_{GM} is also lognormally distributed. This is illustrated in Figure 2 which shows a normal distribution fitted to the ratio $\log(Sa_x/Sa_{GM})$ at $T = 1.0$ sec. The actual and the fitted distribution correspond well. For some ratios the lognormal distribution is a very good approximation; these ratios are indicated with the letter A in the column GMPE in Table 1. For most others the lognormal distribution is a reasonable approximation, which is indicated with the letter B.

From the component definitions that are commonly used for GMPE only the ratio $\log(Sa_{\mathrm{env}}/Sa_{GM})$ is significantly skewed and the lognormal distributed is not suited to describe the distribution of the ratio of the envelope to the geometric mean; this is indicated with the letter C in Table 1. In this case the log ratios are better fitted by a Gamma distribution (Fig. 3).

The skewness becomes even more apparent if a horizontal-component definition other than the geometric mean is used as a reference measure. The ratio $Sa_{\mathrm{env}}/Sa_{\mathrm{random}}$ is used as an example: for a large sample, half of the ratios Sa_{env}/Sa_{random}, by definition, will be equal to 1.0. This is the case when the randomly picked component happens to be the larger of the two components; in the other cases, the ratio Sa_{env}/Sa_{random} will be larger than 1.0. Figure 4 shows the distribution of the latter cases, that is, the distribution of the ratio Sa_{env}/Sa_{random} for ratios >1. Again, the conditional distribution of the log ratio corresponds well to the fitted Gamma distribution. The median of the ratios Sa_{env}/Sa_{random} will be equal to 1.0.

Apart from the ratio Sa_{env}/Sa_{GM} all other ratios with respect to GM_{xy} can be approximated by the lognormal dis-

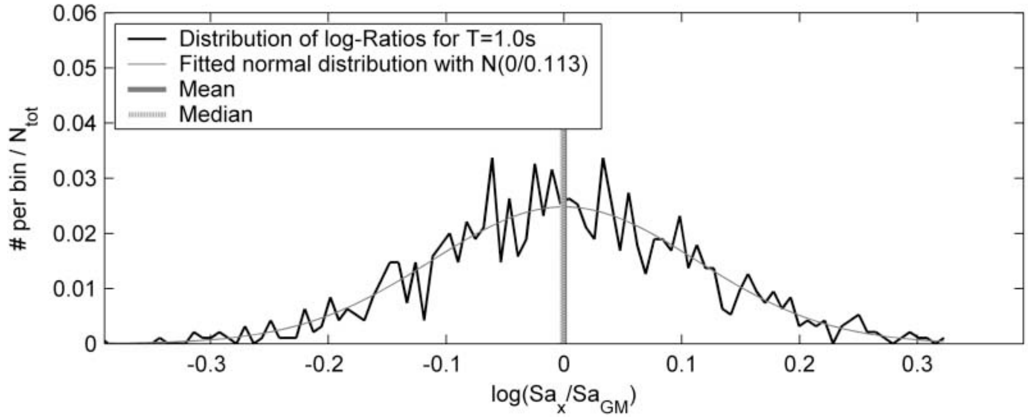

Figure 2. Fitted normal distribution to $\log(Sa_x/Sa_{GMxy})$.

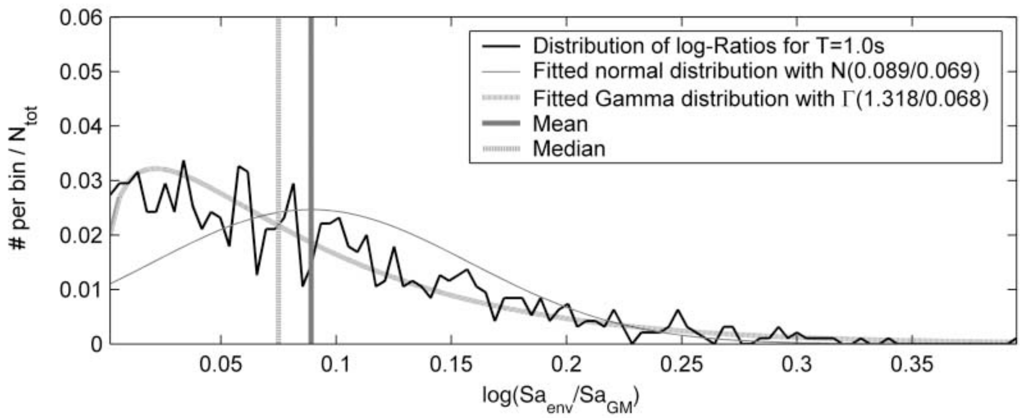

Figure 3. Fitted normal and Gamma distribution to $\log(Sa_{env}/Sa_{GMxy})$.

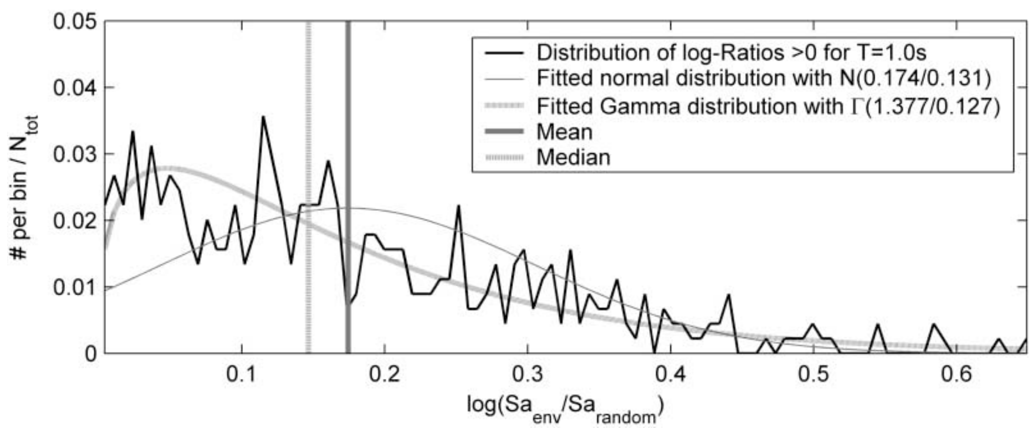

Figure 4. Conditional distributions of Sa_{env}/Sa_{random}: fitted normal and Gamma distribution to $\log(Sa_{env}/Sa_{random})$ for $\log(Sa_{env}/Sa_{random}) > 0$.

tribution. For these, the GMPE equation for Sa_i can be written in the form of equation (5). The methodology for conversion outlined in the following is based on the assumption that the ratio Sa_i/Sa_{GM} can be approximated by a lognormal distribution. For this reason, we advise against applying the methodology to a conversion from Sa_{GM} to Sa_{env} for subsequent use of the converted equation in probabilistic seismic-hazards assessment (PSHA) because, as explained earlier, the hazard calculations are generally based on the assumption of a lognormal distribution of the residuals.

To convert a GMPE from Sa_{GM} to a second measure Sa_i, estimates for the mean and standard deviation of log Sa_i are required. Under the assumption that Sa_i is lognormally distributed, the mean of log Sa_i corresponds to the median of Sa_i. The median of Sa_i can be estimated as

$$\hat{S}a_i = \hat{S}a_{GM} \cdot \left(\frac{Sa_i}{Sa_{GM}}\right)_{\mathrm{median}}, \tag{7}$$

where the median of Sa_{GM} is obtained from the known GMPE and the multiplier is from analysis of strong-motion records. Values of median ratios for different definitions of Sa_i are presented in the next section.

As a second parameter for the adjusted GMPE, the variability of Sa_i has to be estimated. Apart from the aleatory uncertainty of ground motion that is obtained through regression analysis, the variability associated with the estimate of the ratio $\log(Sa_i/Sa_{GM})$ needs to be added as a result of error propagation. The total variance of log Sa_i can hence be written as:

$$\sigma^2_{\mathrm{tot,\,log}\,Sa_i} = \sigma^2_{\log Sa_{GM}}\left(\frac{\sigma_{\log Sa_i}}{\sigma_{\log Sa_{GM}}}\right)^2 + \sigma^2_{\log Sa_i/Sa_{GM}}, \tag{8}$$

where the aleatory variability of log Sa_{GM} is obtained from the known GMPE, the multiplier $\sigma_{\log Sa_i}/\sigma_{\log Sa_{GM}}$ from regression analysis of the dataset for the different definitions of the horizontal component of motion (see section on ratios of sigma values) and the standard deviation of $\log(Sa_i/Sa_{GM})$ by fitting a normal distribution to the log ratios (see the next section).

Median Ratios of Ground-Motion Amplitudes and the Associated Variability

In this section median ratios of Sa_i/Sa_{GM} calculated from the dataset shown in Figure 1 are presented and the variance of $\log(Sa_i/Sa_{GM})$ computed, assuming Sa_i/Sa_{GM} is lognormally distributed. The median ratios are shown in Figure 5. For completeness the median ratios for the different definitions for PGA and PGV are listed in Table 2. Note that figure 2 in Bommer *et al.* (2005) incorrectly displayed mean ratios although labeled as median ratios; this serves as an erratum and Figure 5 thus supersedes the left-hand panel of figure 2 in Bommer *et al.* (2005).

The median ratios of the following component definitions with respect to Sa_{GM} are very close to unity over the entire period range: Sa_x, Sa_y, Sa_{random}, Sa_{both}, Sa_{GMrotD50}, and Sa_{GMrotI50}. The median ratios Sa_{FN}/Sa_{GM} and Sa_{princ1}/Sa_{GM} increase with period range. We believe that this can be attributed to the stronger polarization of ground-motion waves at longer periods. The median ratio of $Sa_{\mathrm{largerPGA}}/Sa_{GM}$ is the only ratio that decreases with period. For $T = 0.0$ sec the component with the larger PGA is equivalent to the envelope of the two components. The longer the period, the less the envelope of the components is correlated to the component with the larger PGA. At long periods almost no correlation occurs between the component with the larger PGA and the component with the greater spectral acceleration, and selecting according to the larger PGA is almost equivalent to choosing randomly. The median ratios of the following definitions with respect to the geometric mean component increase with period: Sa_{env}, Sa_{maxD}, and $Sa_{\mathrm{max\,I}}$. This increase with period implies that the difference between the smaller and the larger spectral ordinate of two orthogonal components increases with period. This is also attributed to the stronger polarization of ground-motion waves at longer periods. Note that the shape of the curve of median ratios of $Sa_{\mathrm{max\,I}}/Sa_{GM}$ depends on the penalty function used for $Sa_{\mathrm{max\,I}}$ and, in particular, on the periods that are included in computing the penalty function.

To simplify the application the variation of the median ratios with period is approximated by simple curves. With exception for the ratio of $Sa_{\mathrm{largerPGA}}/Sa_{GM}$, the approximate equations have the following piecewise linear form in the semilog space:

$$\left(\frac{Sa_i(T_j)}{Sa_{GM}(T_j)}\right)_{\mathrm{median}} = \tag{9}$$

$$\begin{cases} C_1 & T_j \text{ sec} \le 0.15 \text{ sec} \\ C_1 + (C_2 - C_1)\dfrac{\log(T_j/0.15)}{\log(0.8/0.15)} & 0.15 \text{ sec} < T_j < 0.8 \text{ sec} \\ C_2 & 0.8 \text{ sec} \le T_j \le 5.0 \text{ sec} \end{cases}$$

For the ratio of $Sa_{\mathrm{largerPGA}}/Sa_{GM}$ a simple linear relationship is suggested:

$$\frac{Sa_{\mathrm{larger\,PGA}}(T_j)}{Sa_{GM_{xy}}(T_j)} = C_1 + (C_2 - C_1)\frac{T_j}{5.0} \quad T_j \le 5.0 \text{ sec.} \tag{10}$$

The coefficients C_1 and C_2 for the different component definitions are summarized in Table 3. The mathematical form of the piecewise linear approximation is very simple and the agreement between the actual ratios and the approximations can be easily checked visually by sketching the linear relationships onto the figure. Coefficients for fault-normal, fault-parallel, and principal directions are not included because it is recognized that values will vary significantly between records with and without rupture directivity effects. In the

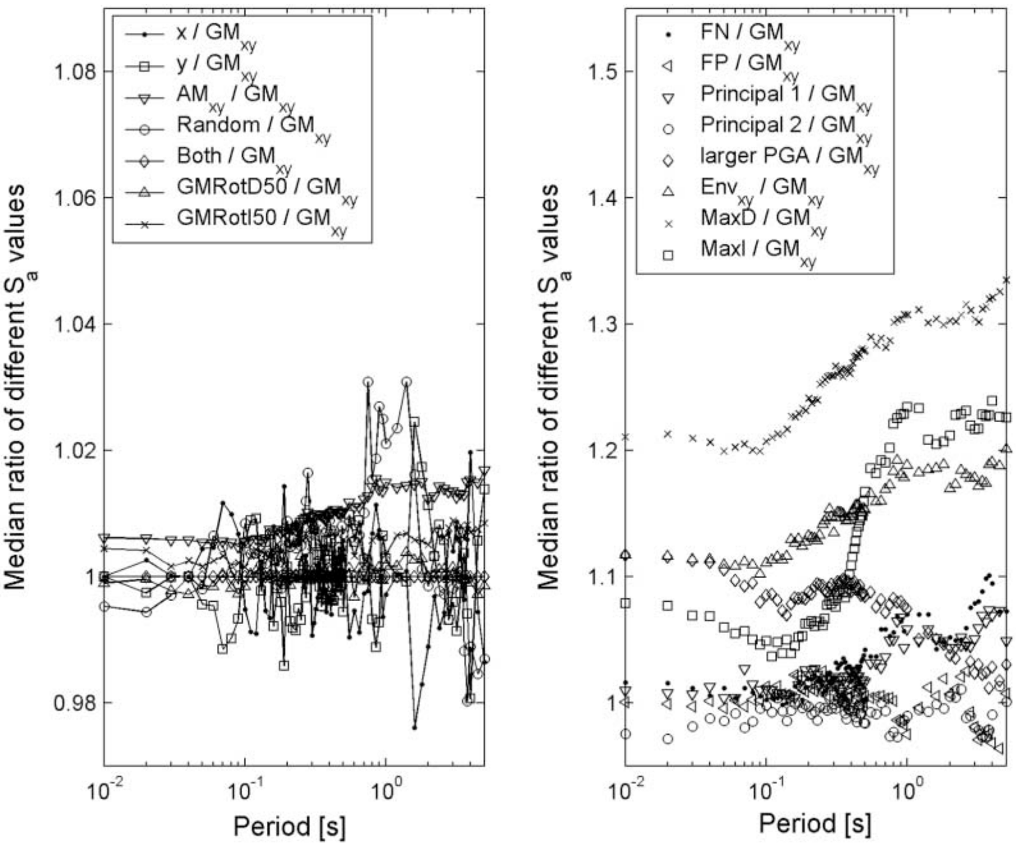

Figure 5. Median ratios of horizontal spectral ordinates for different definitions of the horizontal components of ground motion with the geometric mean of arbitrarily orientated components as reference measure. Note that the y axis of the right-hand panel covers a range of values five times greater than that of the left-hand panel.

Table 2
Median and Standard Deviation of Log Ratios of *PGA* and *PGV* Values and Ratios of the Sigmas Obtained from Regression Analysis for *PGA* and *PGV*

	PGA			PGV		
	Ratio of PGA Values			Ratio R of PGV Values		
	Median	Std. of Log Ratio	Ratio R of σ_{PGA} Values	Median	Std. of Log Ratio	Ratio R of σ_{PGV} Values
x/GM_{xy} resp. y/GM_{xy}	1.00	0.07	1.04	1.00	0.09	1.05
AM_{xy}/GM_{xy}	1.00	0.01	1.00	1.00	0.01	1.00
GMRotD50/GM_{xy}	1.00	0.02	1.00	1.00	0.03	1.00
Random/GM_{xy}	1.00	0.07	1.03	1.00	0.09	1.03
Both/GM_{xy}	1.00	0.07	1.05	1.00	0.09	1.05
Larger PGA/GM_{xy}	1.10	0.05	1.02	1.00	0.06	1.03
Env$_{xy}$/GM_{xy}	1.10	0.05	1.02	1.15	0.06	1.03
MaxD/GM_{xy}	1.20	0.04	1.02	1.25	0.05	1.03

analysis of the records no distinction was made between records with and without directivity effects. Also not included are values for $Sa_{\mathrm{maxI}}/Sa_{GM}$ because they depend greatly on the penalty function used to determine the optimum angle (see Boore *et al.*, 2006).

The standard deviations of the log ratios of spectral ordinates are shown in Figure 6 and the standard deviations of the log ratios of PGA and PGV are listed in Table 1. Figure 6 shows that the standard deviations of the log ratios are smallest for Sa_{AM}/Sa_{GM}, $Sa_{\mathrm{GMrotD50}}/Sa_{GM}$ and $Sa_{\mathrm{GMrotI50}}/Sa_{GM}$. The values of the standard deviations are, in general, however, all quite small when compared with the sigma values

Table 3
Coefficients for Approximative Equations for Median (C_1, C_2) and Standard Deviation (C_3, C_4) of Log Ratios of Different Definitions of Components Obtained from Regression Analysis and Ratios of Sigma Values (R) Approximated by Average Values over Considered Period Range

	C_1	C_2	C_3	C_4	R
x/GM_{xy} resp. y/GM_{xy}	1.00	1.00	0.07	0.11	1.05
AM_{xy}/GM_{xy}	1.00	1.00	0.01	0.02	1.00
GMRotD50/GM_{xy}	1.00	1.00	0.02	0.03	1.00
GMRotI50/GM_{xy}	1.00	1.00	0.03	0.04	1.00
Random/GM_{xy}	1.00	1.00	0.07	0.11	1.05
Both/GM_{xy}	1.00	1.00	0.07	0.11	1.05
Larger PGA/GM_{xy}	1.10	1.00	0.05	0.11	1.04
Env_{xy}/GM_{xy}	1.10	1.20	0.04	0.07	1.02
MaxD/GM_{xy}	1.20	1.30	0.04	0.06	1.02

obtained from regression analysis which are typically between 0.2 and 0.3. Hence, the total standard deviation (see equation 8) will not be significantly greater than $\sigma_{\log Sa_i}$.

Except for $Sa_{\max 1}/Sa_{GM}$, which is not included in Table 3, the variation of the standard deviations with period is very similar to the variations of median ratios with periods. Hence, equations with the same functional form are again used to approximate the standard deviations of the log ratios:

$$\sigma_{\log}(Sa_{GM/Sa_i})(T_j) = \begin{cases} C_3 & T_j \text{ sec} \leq 0.15 \text{ sec} \\ C_3 + (C_4 - C_3)\dfrac{\log(T_j/0.15)}{\log(0.8/0.15)} & 0.15 \text{ sec} < T_j < 0.8 \text{ sec} \\ C_4 & 0.8 \text{ sec} \leq T_j \leq 5.0 \text{ sec} \end{cases} \tag{11}$$

Note that equation (11) was also used to describe the standard deviation of $\log(Sa_{\text{largerPGA}}/Sa_{GM})$. The coefficients C_3 and C_4 for the different component definitions are summarized in Table 3.

Ratios of Sigma Values

For the final part of the adjustment, it is necessary to determine the ratio between the aleatory variabilities that result from using different definitions of the horizontal component. To obtain these estimates, simple GMPEs have been derived by performing regression analyses on the dataset shown in Figure 1, using each of the horizontal-component definitions considered. Note that in figure 2 of Bommer *et al.* (2005) the residuals were calculated using one equation—Abrahamson and Silva (1997) which uses the geometric mean component—in all cases, which is less rigorous than the approach applied here.

With exception of the definition "both components," the sigma values for each horizontal component definition were determined by using a one-stage maximum likelihood regression analysis (Joyner and Boore, 1993). The functional form of the ground-motion prediction equation that was used for the regression analysis included linear and quadratic terms of a magnitude and a geometric spreading term. The sigma values of the ground-motion measure "both components" were determined as the sum of the variance of the ground-motion measure GM_{xy} and the intracomponent variability determined from the difference of the x and y components (Boore, 2005). The aim of the regression analysis was not to derive state-of-the-art GMPEs but to derive, by simple means, a set of GMPEs for different horizontal-component definitions by using the same dataset and with the same regression method. We assume that the ratios of the sigma values are fairly insensitive to assumptions made during the regression analysis even if these do affect the actual sigma values themselves.

The sigma ratios are shown in Figure 7; this figure supersedes the right-hand panel of figure 2 in Bommer *et al.* (2005). Most of the curves in Figure 7, in particular, the sigma ratios $\sigma_{\log Sa_{FN}}/\sigma_{\log Sa_{GM}}$ and $\sigma_{\log princ1}/\sigma_{\log Sa_{GM}}$, show fairly erratic variation over the period range. No physical reason for this behavior could be found and it is assumed that this variation reflects the distribution of the dataset. Therefore, we suggest using values that have been averaged over the period range for analysis. The averaged values are tabulated as variable R in Table 2 (for PGA and PGV) and in Table 3 (for spectral ordinates). All sigma ratios except $\sigma_{\log Sa_{AM}}/\sigma_{\log Sa_{GM}}$, $\sigma_{\log Sa_{\text{GMrotD50}}}/\sigma_{\log Sa_{GM}}$ and $\sigma_{\log Sa_{\text{GMrotI50}}}/\sigma_{\log Sa_{GM}}$ are greater than one. The ratios might not be large but, as pointed out previously, even small differences in σ might have an impact on the results of PSHA in particular if low probabilities of exceedence are considered.

Conclusions and Discussion

A practical approach for the conversion between different definitions of the horizontal component of ground motion has been presented. The approach is based on median ratios of the definitions, the variance of these ratios, and the ratios of standard deviations associated with GMPEs. Empirical values of these ratios have been derived from a large strong-motion dataset, presented in graphical form, and approximated as simple equations for easy application. The geometric mean GM_{xy} has been chosen as reference measure for all ratios because it is the most commonly used definition in current GMPEs.

The ratios have been derived using a dataset of accelerograms from shallow crustal earthquakes and the results have been found relatively insensitive to the actual dataset employed. However, the ratios have not been checked for subduction events or recordings from stable continental regions, and so should be applied with caution to such settings. To explore the applicability of the ratios to such regions could be the subject of future studies, as could exploring variations of the ratios with magnitude, distance, and other explanatory variables. Herein the ratios have assumed to vary only with response period. In addition, note that, for

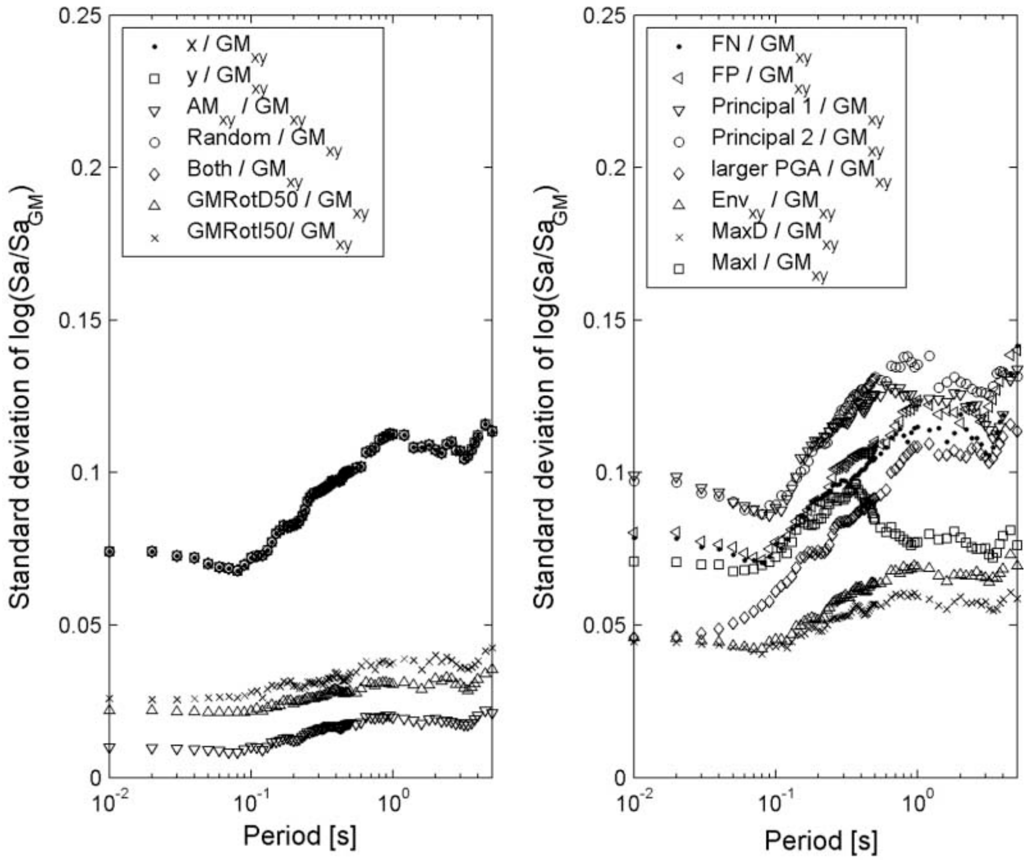

Figure 6. Standard deviation of logarithmic ratios of horizontal spectral ordinates for different definitions of the horizontal components of ground motion with the geometric mean of arbitrarily orientated components as reference measure.

reasons of space, all ratios have been reported with respect to one definition (GM_{xy}); it must be acknowledged that this will result in reduced accuracy when converting between two other definitions. However, in view of current dominance of the geometric mean definition—both in GMPEs and in PSHA—such conversions are unlikely to be required very often.

The results include ratios of median values and ratios of sigma values, plus variances associated with the former. In the strictest sence, the results should also include the variances on the sigma ratios, but these were judged too small to be included, especially because most of the sigma ratios are already very close to unity. By way of illustration, the conversion from GM_{xy} to the MaxD component at a response period of $T = 1.0$ sec is used: with the coefficients derived from the approximate equations, the median of $\hat{S}a_{\mathrm{MaxD}}$ ($T =$ 1.0 sec) would be $\hat{S}a_{GM} \cdot 1.30$ and the total variance of log Sa_{MaxD} would vary as $\sigma^2_{\log Sa_{GM}}(T) \cdot 1.02^2 + 0.06^2$. For the same case as Ambraseys *et al.* (1996) used for illustration of the impact of magnitude conversions in the introductory section, this transformation increases the sigma value from 0.32 to 0.332, with the increase due to equal contributions from sigma ratio and from the propagation of the variability associated with the ratios of the median values. The net impact is therefore of the same order as that of the adjustment for different magnitude scales.

The method of conversion that has been proposed is based on the assumption that both horizontal ground-motion measures and their ratio are longnormally distributed. The results showed that this assumption holds reasonably well for most pairs of horizontal-component definitions, but not for a few; for these other ratios, the Gamma distribution was found to be more appropriate. The most significant case for which the lognormal distribution was found not to fit the residuals is the ratio of the envelope of the two orthogonal response spectra and the geometric mean. This is important because these are currently the two most widely used definitions in ground-motion prediction equations and hence this is precisely the conversion that it is most likely to be required for setting up logic trees for seismic-hazard analysis. Given that it cannot be assumed that the transformed predictive equation would have lognormally distributed residuals, these should, strictly speaking, not be used to provide input to PSHA. This fact, combined with the large penalty in terms of increased aleatory variability as a result of applying these and other adjustments for parameter incompatibilities (Scherbaum *et al.*, 2005), points to the need to establish standard definitions for predicted and explanatory variable

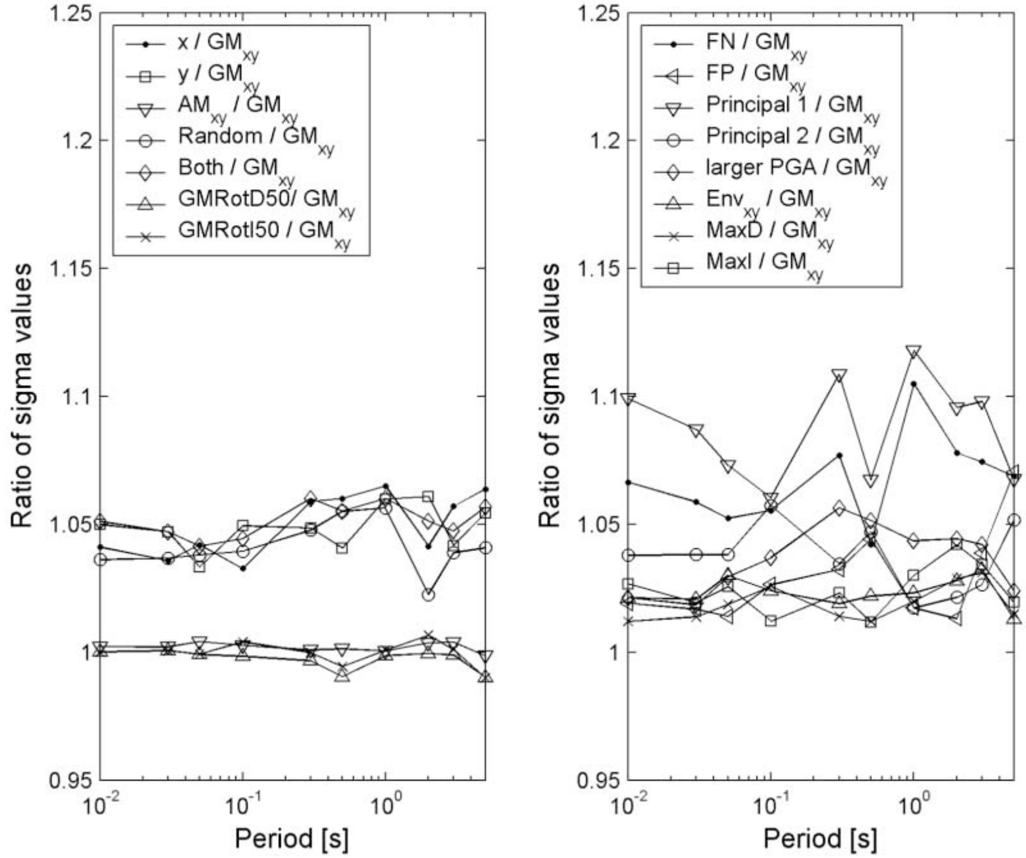

Figure 7. Sigmas obtained from regression analysis for different definitions of the horizontal components of ground motion with the geometric mean of arbitrarily orientated components as reference measure.

to be used in ground-motion prediction equations. The current diversity in the use of different parameter definitions necessitates complicated adjustments when equations are combined in a logic-tree framework (Bommer *et al.*, 2005) and these adjustments lead to unavoidably inflated sigma values.

The formulation presented in this article is focused explicitly on the adjustment of GMPEs using one component definition for use in seismic-hazard analysis where the output is required in terms of another definition. In some cases, conversions between different definitions of the horizontal component of ground motion might be required at a later stage of the project, for example, after a PSHA has been performed and the engineer is faced with a uniform hazard spectrum (UHS). In this case, the UHS for the component definition which was used in the PSHA needs to be converted to a spectrum for a different definition. At this stage, it is no longer possible to separate the median value and aleatory variability in the conversion process and the conversion inevitably becomes more of an approximation. The median ratios as presented in Figure 5 could serve as a first estimate for the multiplier of the spectrum. If from disaggregation of the hazard function an estimate for a representative number, ε, of standard deviations above the median is available a second, and probably better, estimate for the multiplier at each period of the spectrum could be derived as a weighted sum of the median ratio and the sigma ratio. The weighting factors would be the relative contributions of median and above-median ground motion to the spectral ordinate of the original spectrum.

The results of this study are also relevant to the issue of selecting and scaling natural accelerograms for use in dynamic structural analysis. When suites of accelerograms are prepared for such analyses, they will invariably be adjusted in some sense to a target elastic design-response spectrum, hence, the component definition used in the derivation of the latter must be clearly stated, in particular, when bidirectional dynamic input is required. In such cases, the ratios of median values presented in this article can be useful to guide scaling of orthogonal acceleration time series to both match the target-response spectrum and retain the natural differences between the two perpendicular components of motion.

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